Adding a Noise Component To A Color Decomposition Model For Improving Color Texture Extraction

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Abstract

Following the recent work of J-F. Aujol and A. Chambolle, a decomposition model of grayscale images into three components (geometrical, texture and noise) has recently been proposed. Inspired by this work, J-F. Aujol and S. Ha Hung have introduced a new decomposition model for color images. This model splits an image into only two components: a geometrical component and a texture component.

The major contribution of this paper is to add a noise component into the image decomposition model for color images in order to better separate texture from noise. Several numerical exemples illustrate the benefit of our approach.

Introduction

Decomposing an image into meaningful components is an important and challenging inverse problem in image processing.

Y. Meyer has recently introduced [8] a new model to split a given image into two components: a geometrical component and a texture one. Inspired by this work, numerical models have been developed to carry out the decomposition of grayscale images.

In [1], J-F. Aujol and A. Chambolle propose a decomposition model which splits a grayscale image into three components: the first one, $u \in BV$ [1], containing the structure of the image, a second one, $v \in G$ [2], the texture, and the third one, $w \in E$ [3], the noise.

In [2], J-F. Aujol and S. Ha Kang, introduce an algorithm for a color decomposition model which splits a color image into only two components: a geometrical one and a texture one. For this decomposition, they use a generalization of Meyer's G norm applied to RGB vectorial color image, and use Chromaticity and Brightness color model with total variation minimization [4].

In this paper, an extension of the decomposition algorithm for color images is presented. More precisely, the addition of a noise component to the decomposition model of J-F. Aujol and S. Ha Kang [2] is proposed.

This article is organized as follows: in the first section, the litterature about "Different decomposition models" is presented and the following section introduce the "addition of a noise component in the color decomposition model". In section "A new algorithm for the color decomposition model" our new algorithm is exposed. After presenting our numerical experiments, the choice of parameters is explained in the final section.

Different decomposition models

The grayscale decomposition model

In [1], J-F. Aujol and A. Chambolle propose a discretized functionnal for splitting a grayscale image f into a geometrical component u, a texture component v and a noise component w. The decomposition is given by minimizing a functionnal F:

$$\inf_{(u,v,w)\in X^3} F(u,v,w) \tag{1}$$

with

$$F(u,v,w) = J(u) + J^*\left(\frac{v}{\mu}\right) + B^*\left(\frac{w}{\lambda}\right) + \frac{1}{2\alpha}\|f-u-v-w\|_{L^2}$$

where $J(u)$ is the total variation related to the extraction of the geometrical component, $J^*\left(\frac{v}{\mu}\right)$, $B^*\left(\frac{w}{\lambda}\right)$ are the Legendre-Fenchel transforms[4] of respectively J and B [3] for the extraction of texture and noise components, parameter α controls the L^2-norm of the residual $f-u-v-w$ and X is the discrete euclidean space $\mathbb{R}^{N\times N}$ for images of size $N\times N$.

For minimizing this functionnal, Chambolle's projection algoritm is used [1]. The Chambolle's projection P on space λB_G [5] of f is denoted $P_{\lambda B_G}(f)$ and is solved by an iterative algorithm. This algorithm starts with $P^0 = 0$ and for each pixel (i,j) and at each step $n+1$ we have:

$$P^{n+1}_{i,j} = \frac{P^n_{i,j} + \tau\left(\triangle div(P^n) - \frac{f}{\lambda}\right)_{i,j}}{1+\tau\left|\triangle div(P^n) - \frac{f}{\lambda}\right|_{i,j}} \tag{2}$$

In [3] a sufficient condition ensuring the convergence of this algorithm is given: $\tau \leqslant \frac{1}{8}$.

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[1] $BV(\Omega)$ is the subspace of functions $u \in L^1(\Omega)$ such that the following quantity, called the total variation of u, is finite:

$$J(u) = sup\left\{\int_\Omega u(x)div(\xi(x))dx\right\}$$

such that $\xi \in C^1_c(\Omega,\mathbb{R}^2), ||\xi||_{L^\infty(\Omega)} \leq 1$

[2] G is the subspace introduced by Meyer for oscillating patterns. In such a space oscillating patterns have a small norm.

[3] E is another dual space to model oscillating patterns. $\dot{B}^1_{1,1}$ is the usual homogeneous Besov space and the dual space of $\dot{B}^1_{1,1}$ is the Banach space $E = \dot{B}^\infty_{-1,\infty}$

[4] The Legendre-Fenchel transform of F is given by $F^*(v) = sup_u(\langle u,v\rangle_{L^2} - F(u))$, where $\langle .,.\rangle_{L^2}$ stands for the L^2 inner product [9]

[5] $\lambda B_G = \{f \in G/\|f\|_G \leqslant \lambda\}$

To solve (1), the authors propose to solve successively three different minimization problems.

At step n :

1. u and v have been previously computed, we estimate:

$$\tilde{w} = P_{\delta B_E}(f - u - v)$$

2. then we compute:

$$\tilde{v} = P_{\mu B_G}(f - u - \tilde{w})$$

3. and we finally obtain:

$$\tilde{u} = f - u - \tilde{v} - \tilde{w} - P_{\lambda B_G}(f - \tilde{v} - \tilde{w})$$

This operation is repeated until :

$$max(|\tilde{u} - u|, |\tilde{v} - v|, |\tilde{w} - w|) \leqslant \varepsilon$$

In [1], the authors replace $P_{\delta B_E}(f - u - v)$ by $f - u - v - W_{ST}(f - u - v, \delta)$ where $W_{ST}(f - u - v, \delta)$ stands for the wavelet soft-thresholding of $f - u - v$ with threshold δ defined by :

$$S_\delta\left(d_i^j\right) = \left\{ \begin{array}{ll} d_i^j - \delta sign\left(d_i^j\right) & if \quad |d_i^j| > \delta \\ 0 & if \quad |d_i^j| \leqslant \delta \end{array} \right. \tag{3}$$

where d_i^j is the wavelet coefficient, j the resolution and $i \in \{x, y, xy\}$.

Color decomposition model

For decomposing color images, J-F. Aujol and S. Ha Kang propose the following functional [2]:

$$\inf_{u,v} \left\{ J(u) + J^*\left(\frac{v}{\mu}\right) + \frac{1}{2\lambda}\|f - u - v\|_{L^2} \right\} \tag{4}$$

The higher λ is, the more negligible the residual $f - u - v$ is. μ controls the $\|.\|_G$ norm of v.

For solving this functional, the authors proceed as follows :

1. v being fixed, we search for u as a solution of :

$$\inf_{u} \left(J(u) + \frac{1}{2\lambda}\|f - u - v\|^2 \right)$$

2. Then, u being fixed, we search for v as a solution of :

$$\inf_{v} \|f - u - v\|^2$$

This operation is repeated until :

$$max(|\tilde{u} - u|, |\tilde{v} - v|) \leqslant \xi$$

with ξ a given threshold. To solve these two minimization problems, the direct total variation minimization approach is used[5], since the Chambolle's projection only works for grayscale images.

The aim of our work is to add a noise component to this color decomposition model in order to better seperate texture from noise.

Adding a noise component to the color decomposition model

To remove noise, a large variety of approaches have been proposed in the litterature, including wavelet shrinkage and non linear diffusion filtering. We use a connection given by Weickert [11] between these two techniques to introduce the noise component in the image decomposition model.

Let us now focus our attention on color image wavelet shrinkage. In the litterature, the different color channels are frequently shrunk marginally. This leads to create artifacts at color edges. On the contrary for nonlinear diffusion filtering of color images, a process with a joint diffusivity that steers the evolution of all three channels is often used [7].

By considering an explicit discretisation and relating it to wavelet shrinkage, Weickert gives shrinkage rules where all channels are coupled [11]. In contrast to classical shrinkage (3) where the wavelet coefficient are skrunken seperately, this leads to novel shrinkage rules where the wavelet coefficients are coupled :

$$S_\delta\left(d_i^j\right) = d_i^j \left(1 - 4\delta \frac{1}{\sqrt{d_x^j + d_y^j + 2d_{xy}^j}} \right), \forall i \in \{x, y, xy\}$$

To steer the evolution of all three channels, the following shrinkage function S_δ for the wavelet coefficient d_i^{cj} ($c \in \{r, g, b\}$) is proposed :

$$S_\delta\left(d_i^{cj}\right) = d_i^{cj} \left(1 - 12\delta \frac{1}{\sqrt{\sum\limits_{o \in \{r,g,b\}} \left(d_x^{oj} + d_y^{oj} + 2d_{xy}^{oj}\right)}} \right) \tag{5}$$

with $\forall i \in \{x, y, xy\}$ and $\forall c \in \{r, g, b\}$.

If this function is used in the algorithm of J-F. Aujol and S. Ha Kang, a third component, the noise is obtained. In the following section, this shrinkage function is named $W_{ST}^{rgb}()$.

A new algorithm for the color decomposition model

Our algorithm for splitting a color image f into a geometrical component u, a texture component v and a noise component w is the following :

(1) Initialization of f, u, v, w where f_0 is the original image

$f = f_0, \quad u^1 = f_0$

$v^1 = 0 \quad w^1 = W_{ST}^{\prime rgb}(f)$

(2) Iterate N times or until $max(|u^{n+1} - u^n|, |v^{n+1} - v^n|, |w^{n+1} - w^n|) \leqslant \xi$

(a) Seperate f, u^n, v^n, w^n into brightness (f_b, u_b, v_b, w_b) and chromaticity (f_c, u_c, v_c, w_c) components

$f_b = \|f\| \qquad f_c = \dfrac{f}{\|f\|}$

$u_b = \|u^n\| \qquad u_c = \dfrac{u^n}{\|u^n\|}$

$$v_b = \|v^n\| \qquad v_c = \frac{v^n}{\|v^n\|}$$
$$w_b = \|w^n\| \qquad w_c = \frac{w^n}{\|w^n\|}$$

(b) Iterate on k between 1 to M and update u_c and u_b

$$u_c^{k+1} = \mathscr{F}_\alpha^{\varepsilon,\lambda_c}\left(u_c^k, f_c - v_c - w_c\right)$$
$$u_b^{k+1} = \mathscr{F}_\alpha^{\varepsilon,\lambda_b}\left(u_b^k, f_b - v_b - w_b\right)$$
$$u_c^k = u_c^{k+1} \text{ et } u_b^k = u_b^{k+1}$$

(c) Update u and calculate the residual r

$$u^{n+1} = u_c^k * u_b^k$$
$$r^n = f - u^{n+1} - v^n - w^n$$

(d) Iterate for k between 1 to M for updating r

$$r^{k+1} = \mathscr{F}_\alpha^{\varepsilon,\mu}\left(r^k, f - u^{n+1} - v^n - w^n\right)$$
$$r^k = r^{k+1}$$

(e) Update v and w

$$v^{n+1} = f - u^{n+1} - r^{n+1} - w^n$$
$$w^{n+1} = f - u^{n+1} - v^{n+1} - W_{ST}^{'rgb}(f - u^{n+1} - v^{n+1})$$

(f) Preparation for the next iteration

$$u^n = u^{n+1}$$
$$v^n = v^{n+1}$$
$$w^n = w^{n+1}$$

with :

$$\mathscr{F}_\alpha^{\varepsilon,\lambda}\left(u, u^0\right) = \sum_{\beta_j} h_{\alpha\beta_j}^{\varepsilon,\lambda}(u) u_{\beta_j} + h_{\alpha\alpha}^{\varepsilon,\lambda}(u) u_\alpha^0$$

$$h_{\alpha\beta}^{\varepsilon,\lambda}(u) = \frac{w_{\alpha\beta}^{\varepsilon}(u)}{\lambda + \sum_{\gamma\sim\alpha} w_{\alpha\gamma}^{\varepsilon}(u)}$$

$$h_{\alpha\alpha}^{\varepsilon,\lambda}(u) = \frac{\lambda}{\lambda + \sum_{\gamma\sim\alpha} w_{\alpha\gamma}^{\varepsilon}(u)}$$

$$w_{\alpha\beta}^{\varepsilon}(u) = \frac{1}{\sqrt{|\nabla_\alpha u|^2 + \varepsilon^2}} + \frac{1}{\sqrt{|\nabla_\beta u|^2 + \varepsilon^2}}$$

where α, β and γ are neighbouring pixels of u. For more details, see [5].
and :

$$W_{ST}^{'rgb}(f) = \frac{1}{5} \quad \big(W_{ST}^{rgb}(f(x-1,y)) + W_{ST}^{rgb}(f(x+1,y)) + W_{ST}^{rgb}(f(x,y)) + W_{ST}^{rgb}(f(x,y-1)) + W_{ST}^{rgb}(f(x,y+1))\big)$$

for $W_{ST}^{rgb}()$, refer to section "Adding a noise component to the color decomposition model". An averaging of the results for four shifted possibilities is computed in order to avoid block effects.

Numerical experiments

Figure 1 presents numerical results of image decompositions obtained with the algorithm of Aujol and Kang and our new model. The following parameters values have been used in our experiments:

$\delta = 2$ $\quad \mu = 0.1$
$\lambda_c = 0.01$ $\quad N = 2$
$\lambda_b = 0.05$ $\quad M = 30$

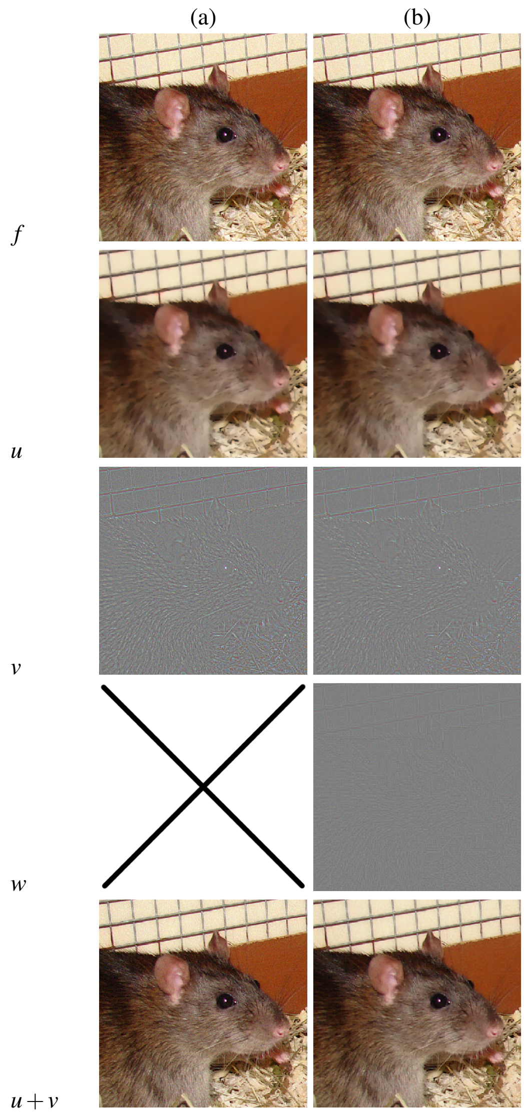

Figure 1. *An image decomposition with two models : column (a) model of J-F. Aujol and S. Ha Kang and column (b) our model*

By observing the recomposed image $u+v$ for each color decomposition model, we can note that our decomposition model smoothes better the homegeneous zones. Moreover, figure 2 shows that the recomposed image contains less noise with our model.

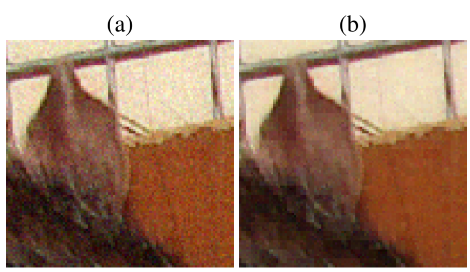

Figure 2. *Detailed view of the recomposed image $u+v$ for the two models : column (a) model of J-F. Aujol and S. Ha Kang and column (b) our model*

A similar observation can be done on figure 3. It represents the error between the original image without noise and the recomposed image $u+v$ using the two models (δ on the absciss).

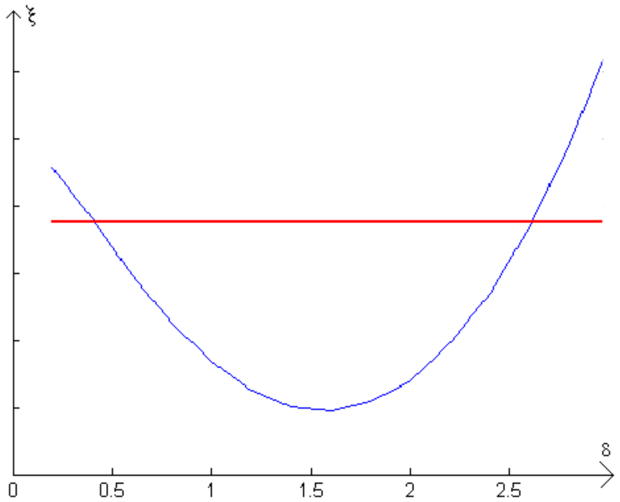

Figure 3. *Error plots between the original image without noise and the recomposed image $u+v$ using UV model (in red) and using UVW model (in blue) with the variation of δ on the absciss*

Two things can be noted on these plots :

- the error between the original image and the recomposed image $u+v$ of Aujol and Kang is always the same, because this model does not take the noise into account. This error is used as a reference for the comparison with our model.
- the error between the original image and our color decomposition model starts by decreasing and is followed by an increase. The decrease is due to the subtraction of noise to the noisy image so that $u+v$ approaches the original image. The increase of the error is caused by an over extraction of noise: a part of the image texture is added to the noise component. The reconstruction $u+v$ and the original image do not match anymore.

Choice of Parameters

In this section, the parameter values are discuted.

Fig. 4 shows that the smaller the couple (λ_c, λ_b), the higher the regularization of u. This parameters rule the strengh of smoothing.

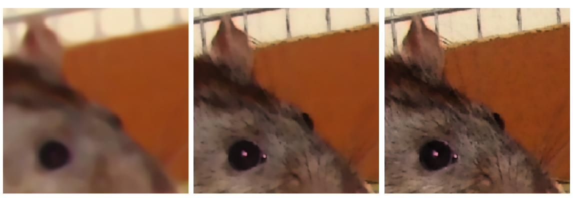

Figure 4. *From left to right : increasing values of couple (λ_c, λ_b)*

Fig. 5 shows that the smaller μ is, the higher the extraction of texture in v. This parameter is very sensitive as it can mix the texture part and the noise part.

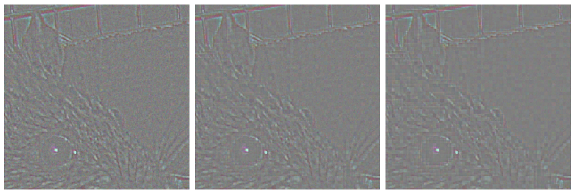

Figure 5. *From left to right : increasing values of μ*

Fig. 6 shows that the higher δ is, the higher the extraction of the noise will be. This parameter is also crucial as if too high, the texture of the image can be extracted in the noise component: it is a trade-off between parameters μ and δ.

Figure 6. *From left to right: increasing values of δ*

Conclusion

In this paper, a new decomposition algorithm has been presented, which can be applied for color image denoising. It combines the total variation minimization model of J-F. Aujol and S. Ha Kang for image decomposition, with a new shrinkage function for the wavelet coefficients.

In our color decomposition model, Haar analysis filters are used. Others filters could improve the results for image denoising by potentially reducing block artifacts. However in this case formula (5) should be changed.

This method can be extended to the temporal domain in order to process videos. If the color decomposition model is applied directly on each image, only the spatial information can be extracted. Future prospects are to extend the analysis to time, leading to a better extraction of spatial and temporal contents. This improved decomposition could be promising for analyzing and characterizing spatio-temporal patterns such as dynamic textures [6].

References

[1] J-F. Aujol and A. Chambolle, *Dual norms and image decomposition models*, International Journal of Computer Vision, volume 63, number 1, pages 85-104, June 2005.

[2] J-F. Aujol and S. Ha Kang, *Color image decomposition and restoration*, Journal of Visual Communication and Image Representation, volume 17, number 4, pages 916-928, August 2006.

[3] A. Chambolle, *An algorithm for Total Variation Minimization ans applications*, Journal of Mathematical Imaging and Vision 20 (2004) 89-97

[4] T.F. Chan, S. Ha Kang and J. Shen, *Total variation denoising and enhancement of color images based on the CB and HSV color models*

[5] T.F. Chan, S. Osher and J. Shen, *The digital TV filter and nonlinear denoising*, IEEE Transactions on image processing, 10:231-241,2001

[6] D. Chetverikov and R. Péteri, *A brief survey of dynamic texture description and recognition*, Proceedings of 4th International Conference on Computer Recognition Systems CORES'05, Advances in Soft Computing, Springer-Verlag, pp. 17-26, Rydzyna, Poland, 2005.

[7] G. Gerig, O. Kubler, R. Kikinis and F. Jolesz, *Nonlinear anisotropic filtering of MRI data*, IEEE Transactions on Medical Imaging, 11:221-232, 1992.

[8] Y. Meyer, *Oscillating patterns in image processing and nonlinear evolution equations*, The fifteenth dean jacqueline B. Lewis Memorial Lectures, Vol. 22 of University Lecture Series, AMS, Providence, 2001.

[9] J-L. Starck, M. ELad, D. Donoho, *Image Decomposition via the Combination of Sparse Representation and Variationnal Approach*, IEEE Trans. Image Process, 2005

[10] G. Steidl, J. Weickert, T. Brox, P. Mrzek, and M. Welk, *On the equivalence of soft wavelet shrinkage, total variation diffusion, total variation regularization, and sides*, Tech. Rep. 26, Department of Mathematics, University of Bremen, Germany, 2003.

[11] J. Weickert, G. Steidl, P. Mrazek, M. Welk and T. Brox, *Diffusion filters and wavelets: What can they learn from each other?*, Chapter 1 of "Handbook of Mathematical Models in Computer Vision".

[12] M. Welk, J. Weickert, G. Steidl, *A Four-Pixel Scheme for Singular Differential Equations*, ScaleSpace05(610-621).